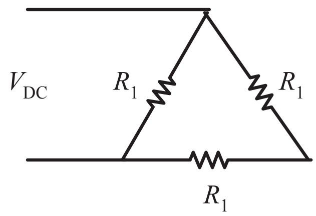
Chapter 7: *Induction Motors*

7-1. A dc test is performed on a 460-V Δ-connected 100-hp induction motor. If V_{DC} = 21 V and I_{DC} = 72 A, what is the stator resistance R_1? *Why is this so?*

SOLUTION If this motor's armature is connected in delta, then there will be two phases in parallel with one phase between the lines tested.



Therefore, the stator resistance R_1 will be

$$\frac{V_{DC}}{I_{DC}} = \frac{R_1(R_1+R_1)}{R_1+(R_1+R_1)} = \frac{2}{3}R_1$$

$$R_1 = \frac{3}{2}\frac{V_{DC}}{I_{DC}} = \frac{3}{2}\left(\frac{21\text{ V}}{72\text{ A}}\right) = 0.438\ \Omega$$

7-2. A 220-V three-phase six-pole 50-Hz induction motor is running at a slip of 3.5 percent. Find:

(a) The speed of the magnetic fields in revolutions per minute

(b) The speed of the rotor in revolutions per minute

(c) The slip speed of the rotor

(d) The rotor frequency in hertz

SOLUTION

(a) The speed of the magnetic fields is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{6} = 1000\text{ r/min}$$

(b) The speed of the rotor is

$$n_m = (1-s)\ n_{sync} = (1-0.035)(1000\text{ r/min}) = 965\text{ r/min}$$

(c) The slip speed of the rotor is

$$n_{slip} = sn_{sync} = (0.035)(1000\text{ r/min}) = 35\text{ r/min}$$

(d) The rotor frequency is

$$f_r = \frac{n_{slip}P}{120} = \frac{(35\text{ r/min})(6)}{120} = 1.75\text{ Hz}$$

7-3. Answer the questions in Problem 7-2 for a 480-V three-phase four-pole 60-Hz induction motor running at a slip of 0.025.

SOLUTION

(a) The speed of the magnetic fields is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{4} = 1800\text{ r/min}$$

(b) The speed of the rotor is

$$n_m = (1-s)\ n_{\text{sync}} = (1-0.025)(1800\text{ r/min}) = 1755\text{ r/min}$$

(c) The slip speed of the rotor is

$$n_{\text{slip}} = s n_{\text{sync}} = (0.025)(1800\text{ r/min}) = 45\text{ r/min}$$

(d) The rotor frequency is

$$f_r = \frac{n_{\text{slip}} P}{120} = \frac{(45\text{ r/min})(4)}{120} = 1.5\text{ Hz}$$

7-4. A three-phase 60-Hz induction motor runs at 715 r/min at no load and at 670 r/min at full load.

(a) How many poles does this motor have?

(b) What is the slip at rated load?

(c) What is the speed at one-quarter of the rated load?

(d) What is the rotor's electrical frequency at one-quarter of the rated load?

SOLUTION

(a) This machine has 10 poles, which produces a synchronous speed of

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{10} = 720\text{ r/min}$$

(b) The slip at rated load is

$$s = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} \times 100\% = \frac{720-670}{720} \times 100\% = 6.94\%$$

(c) The motor is operating in the linear region of its torque-speed curve, so the slip at ¼ load will be

$$s = 0.25(0.0694) = 0.0174$$

The resulting speed is

$$n_m = (1-s)\ n_{\text{sync}} = (1-0.0174)(720\text{ r/min}) = 707\text{ r/min}$$

(d) The electrical frequency at ¼ load is

$$f_r = s f_e = (0.0174)(60\text{ Hz}) = 1.04\text{ Hz}$$

7-5. A 50-kW 440-V 50-Hz two-pole induction motor has a slip of 6 percent when operating at full-load conditions. At full-load conditions, the friction and windage losses are 520 W, and the core losses are 500 W. Find the following values for full-load conditions:

(a) The shaft speed n_m

(b) The output power in watts

(c) The load torque τ_{load} in newton-meters

(d) The induced torque τ_{ind} in newton-meters

(e) The rotor frequency in hertz

SOLUTION

(a) The synchronous speed of this machine is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{2} = 3000\text{ r/min}$$

Therefore, the shaft speed is

$$n_m = (1-s)\ n_{\text{sync}} = (1-0.06)(3000\text{ r/min}) = 2820\text{ r/min}$$

(b) The output power in watts is 50 kW (stated in the problem).

(c) The load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{50\text{ kW}}{(2820\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 169.3\text{ N}\cdot\text{m}$$

(d) The induced torque can be found as follows:

$$P_{\text{conv}} = P_{\text{OUT}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{misc}} = 50\text{ kW} + 520\text{ W} + 500\text{ W} = 51.2\text{ kW}$$

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{51.2\text{ kW}}{(2820\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 173.4\text{ N}\cdot\text{m}$$

(e) The rotor frequency is

$$f_r = sf_e = (0.06)(50\text{ Hz}) = 3.00\text{ Hz}$$

7-6. A three-phase 60-Hz two-pole induction motor runs at a no-load speed of 3580 r/min and a full-load speed of 3440 r/min. Calculate the slip and the electrical frequency of the rotor at no-load and full-load conditions. What is the speed regulation of this motor [Equation (4-57)]?

SOLUTION The synchronous speed of this machine is 3600 r/min. The slip and electrical frequency at no-load conditions is

$$s_{\text{nl}} = \frac{n_{\text{sync}} - n_{\text{nl}}}{n_{\text{sync}}} \times 100\% = \frac{3600-3580}{3600} \times 100\% = 0.56\%$$

$$f_{r,\text{nl}} = sf_e = (0.0056)(60\text{ Hz}) = 0.33\text{ Hz}$$

The slip and electrical frequency at full load conditions is

$$s_{\text{fl}} = \frac{n_{\text{sync}} - n_{\text{nl}}}{n_{\text{sync}}} \times 100\% = \frac{3600-3440}{3600} \times 100\% = 4.44\%$$

$$f_{r,\text{fl}} = sf_e = (0.0444)(60\text{ Hz}) = 2.67\text{ Hz}$$

The speed regulation is

$$\text{SR} = \frac{n_{\text{nl}} - n_{\text{fl}}}{n_{\text{fl}}} \times 100\% = \frac{3580-3440}{3440} \times 100\% = 4.1\%$$

7-7. A 208-V four-pole 60-Hz Y-connected wound-rotor induction motor is rated at 15 hp. Its equivalent circuit components are

$R_1 = 0.220\ \Omega$ $\quad R_2 = 0.127\ \Omega$ $\quad X_M = 15.0\ \Omega$

$X_1 = 0.430\ \Omega$ $\quad X_2 = 0.430\ \Omega$

$P_{\text{mech}} = 300\text{ W}$ $\quad P_{\text{misc}} \approx 0$ $\quad P_{\text{core}} = 200\text{ W}$

For a slip of 0.05, find

(a) The line current I_L

(b) The stator copper losses P_{SCL}

(c) The air-gap power P_{AG}

(d) The power converted from electrical to mechanical form P_{conv}

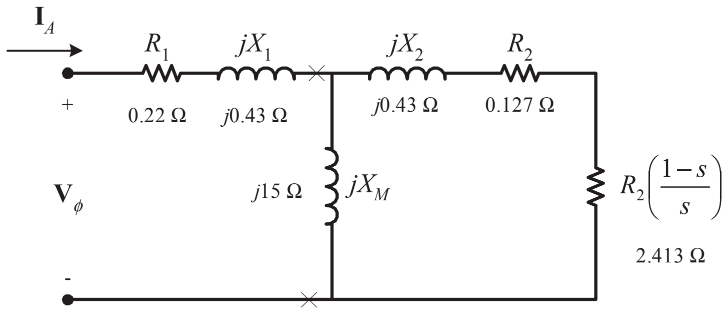
(e) The induced torque τ_{ind}

(f) The load torque τ_{load}

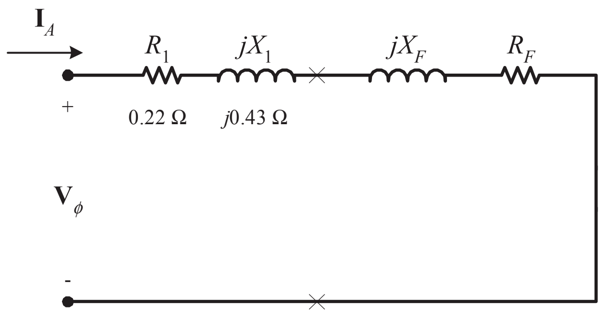
(g) The overall machine efficiency

(h) The motor speed in revolutions per minute and radians per second

SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j15\,\Omega} + \frac{1}{2.54 + j0.43}} = 2.337 + j0.803 = 2.47\angle 19^\circ\ \Omega$$

The phase voltage is $208/\sqrt{3}$ = 120 V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0° \text{ V}}{0.22\,\Omega + j0.43\,\Omega + 2.337\,\Omega + j0.803\,\Omega}$$

$$I_L = I_A = 42.3\angle -25.7° \text{ A}$$

(b) The stator copper losses are

$$P_{\text{SCL}} = 3I_A^{\ 2}R_1 = 3(42.3 \text{ A})^2(0.22\ \Omega) = 1180 \text{ W}$$

(c) The air gap power is $P_{\text{AG}} = 3I_2^{\ 2}\dfrac{R_2}{s} = 3I_A^{\ 2}R_F$

(Note that $3I_A^{\ 2}R_F$ is equal to $3I_2^{\ 2}\dfrac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2 / s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{\text{AG}} = 3I_2^{\ 2}\frac{R_2}{s} = 3I_A^{\ 2}R_F = 3(42.3 \text{ A})^2(2.337\ \Omega) = 12.54 \text{ kW}$$

(d) The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1-s)P_{\text{AG}} = (1-0.05)(12.54 \text{ kW}) = 11.92 \text{ kW}$$

(e) The induced torque in the motor is

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} = \frac{12.54 \text{ kW}}{(1800 \text{ r/min})\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)} = 66.5 \text{ N}\cdot\text{m}$$

(f) The output power of this motor is

$$P_{\text{OUT}} = P_{\text{conv}} - P_{\text{mech}} - P_{\text{core}} - P_{\text{misc}} = 11.92 \text{ kW} - 300 \text{ W} - 200 \text{ W} - 0 \text{ W} = 11.42 \text{ kW}$$

The output speed is

$$n_m = (1-s)\,n_{\text{sync}} = (1-0.05)(1800 \text{ r/min}) = 1710 \text{ r/min}$$

Therefore the load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{11.42 \text{ kW}}{(1710 \text{ r/min})\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)} = 63.8 \text{ N}\cdot\text{m}$$

(g) The overall efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{P_{\text{OUT}}}{3V_\phi I_A \cos\theta} \times 100\%$$

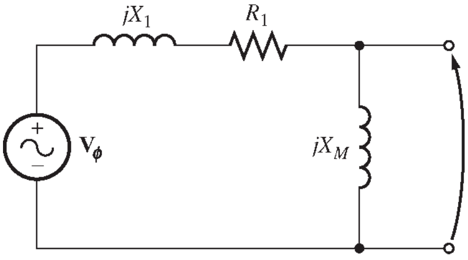
$$\eta = \frac{11.42 \text{ kW}}{3(120 \text{ V})(42.3 \text{ A})\cos 25.7°} \times 100\% = 83.2\%$$

(h) The motor speed in revolutions per minute is 1710 r/min. The motor speed in radians per second is

$$\omega_m = (1710 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 179 \text{ rad/s}$$

7-8. For the motor in Problem 7-7, what is the slip at the pullout torque? What is the pullout torque of this motor?

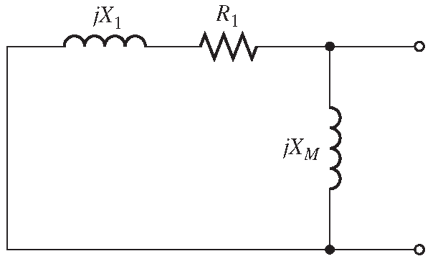
SOLUTION The slip at pullout torque is found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model.



$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + jX_1 + jX_M}\mathbf{V}_\phi$$

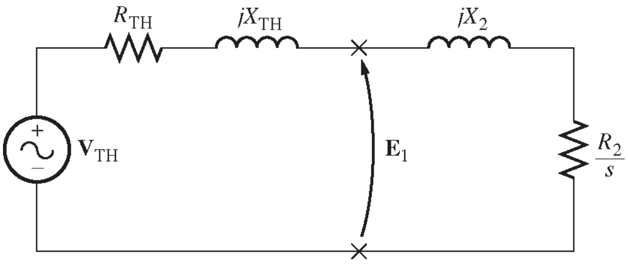
$$V_{TH} = \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}\mathbf{V}_\phi$$

(a)



$$Z_{TH} = \frac{jX_M (R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

(b)



(c)

$$Z_{TH} = \frac{jX_M\left(R_1 + jX_1\right)}{R_1 + j\left(X_1 + X_M\right)} = \frac{(j15\ \Omega)(0.22\ \Omega\ + j0.43\ \Omega)}{0.22\ \Omega\ + j(0.43\ \Omega + 15\ \Omega)} = 0.208 + j0.421\ \Omega = 0.470\angle 63.7^\circ\ \Omega$$

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + j(X_1 + X_M)}\mathbf{V}_\phi = \frac{(j15\,\Omega)}{0.22\,\Omega + j(0.43\,\Omega + 15\,\Omega)}(120\angle 0^\circ \text{ V}) = 116.7\angle 0.8^\circ \text{ V}$$

The slip at pullout torque is

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^{\ 2} + \left(X_{TH} + X_2\right)^2}}$$

$$s_{max} = \frac{0.127\ \Omega}{\sqrt{(0.208\ \Omega)^2 + (0.421\ \Omega\ + 0.430\ \Omega)^2}} = 0.145$$

The pullout torque of the motor is

$$\tau_{\max} = \frac{3V_{\text{TH}}^2}{2\omega_{\text{sync}}\left[R_{\text{TH}} + \sqrt{R_{\text{TH}}^2 + \left(X_{\text{TH}} + X_2\right)^2}\right]}$$

$$\tau_{\max} = \frac{3(116.7\text{ V})^2}{2(188.5\text{ rad/s})\left[0.208\ \Omega + \sqrt{(0.208\ \Omega)^2 + (0.421\ \Omega\ + 0.430\ \Omega)^2}\right]}$$

$$\tau_{\max} = 100\text{ N}\cdot\text{m}$$

7-9. *(a)* Calculate and plot the torque-speed characteristic of the motor in Problem 7-7. *(b)* Calculate and plot the output power versus speed curve of the motor in Problem 7-7.

SOLUTION

(a) A MATLAB program to calculate the torque-speed characteristic is shown below.

```
% M-file: prob7_9a.m
% M-file create a plot of the torque-speed curve of the
%   induction motor of Problem 7-7.

% First, initialize the values needed in this program.
r1 = 0.220;                % Stator resistance
x1 = 0.430;                % Stator reactance
r2 = 0.127;                % Rotor resistance
x2 = 0.430;                % Rotor reactance
xm = 15.0;                 % Magnetization branch reactance
v_phase = 208 / sqrt(3);   % Phase voltage
n_sync = 1800;             % Synchronous speed (r/min)
w_sync = 188.5;            % Synchronous speed (rad/s)

% Calculate the Thevenin voltage and impedance from Equations
% 7-38 and 7-41.
v_th = v_phase * ( xm / sqrt(r1^2 + (x1 + xm)^2) );
z_th = ((j*xm) * (r1 + j*x1)) / (r1 + j*(x1 + xm));
r_th = real(z_th);
x_th = imag(z_th);

% Now calculate the torque-speed characteristic for many
% slips between 0 and 1.  Note that the first slip value
% is set to 0.001 instead of exactly 0 to avoid divide-
% by-zero problems.
s = (0:1:50) / 50;          % Slip
s(1) = 0.001;
nm = (1 - s) * n_sync;      % Mechanical speed

% Calculate torque versus speed
for ii = 1:51
   t_ind(ii) = (3 * v_th^2 * r2 / s(ii)) / ...
           (w_sync * ((r_th + r2/s(ii))^2 + (x_th + x2)^2) );
end

% Plot the torque-speed curve
figure(1);
plot(nm,t_ind,'k-','LineWidth',2.0);
xlabel('\bf\itn_{m}');
ylabel('\bf\tau_{ind}');
```

$$I_L = \frac{V}{\sqrt{\left(R_S + R_L\right)^2 + \left(X_S + X_L\right)^2}}$$

The power supplied to the load is

$$P = I_L^{\,2} R_L = \frac{V^2 R_L}{\left(R_S + R_L\right)^2 + \left(X_S + X_L\right)^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{\left[\left(R_S + R_L\right)^2 + \left(X_S + X_L\right)^2\right] V^2 - V^2 R_L \left[2\left(R_S + R_L\right)\right]}{\left[\left(R_S + R_L\right)^2 + \left(X_S + X_L\right)^2\right]^2}$$

To find the point of maximum power supplied to the load, set $\partial P / \partial R_L = 0$ and solve for R_L.

$$\left[\left(R_S + R_L\right)^2 + \left(X_S + X_L\right)^2\right] V^2 - V^2 R_L \left[2\left(R_S + R_L\right)\right] = 0$$

$$\left[\left(R_S + R_L\right)^2 + \left(X_S + X_L\right)^2\right] = 2R_L \left(R_S + R_L\right)$$

$$R_S^{\,2} + 2R_S R_L + R_L^{\,2} + \left(X_S + X_L\right)^2 = 2R_S R_L + 2R_L^{\,2}$$

$$R_S^{\,2} + R_L^{\,2} + \left(X_S + X_L\right)^2 = 2R_L^{\,2}$$

$$R_S^{\,2} + \left(X_S + X_L\right)^2 = R_L^{\,2}$$

Therefore, for maximum power transfer, the load resistor should be

$$\boxed{R_L = \sqrt{R_S^{\,2} + \left(X_S + X_L\right)^2}}$$

7-14. A 440-V 50-Hz six-pole Y-connected induction motor is rated at 75 kW. The equivalent circuit parameters are

$R_1 = 0.082\ \Omega$	$R_2 = 0.070\ \Omega$	$X_M = 7.2\ \Omega$
$X_1 = 0.19\ \Omega$	$X_2 = 0.18\ \Omega$	
$P_{\text{F\&W}} = 1.3\text{ kW}$	$P_{\text{misc}} = 150\text{ W}$	$P_{\text{core}} = 1.4\text{ kW}$

For a slip of 0.04, find

(a) The line current I_L

(b) The stator power factor

(c) The rotor power factor

(d) The stator copper losses P_{SCL}

(e) The air-gap power P_{AG}

(f) The power converted from electrical to mechanical form P_{conv}

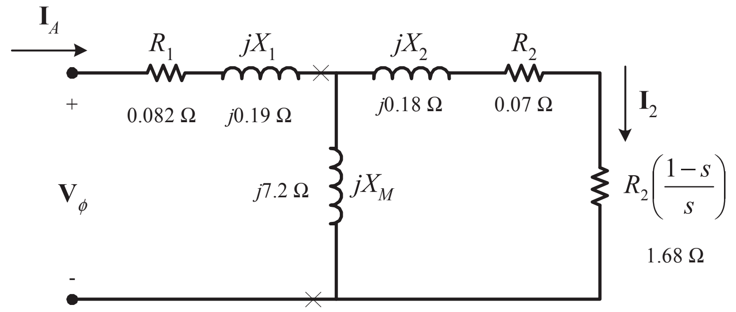
(g) The induced torque τ_{ind}

(h) The load torque τ_{load}

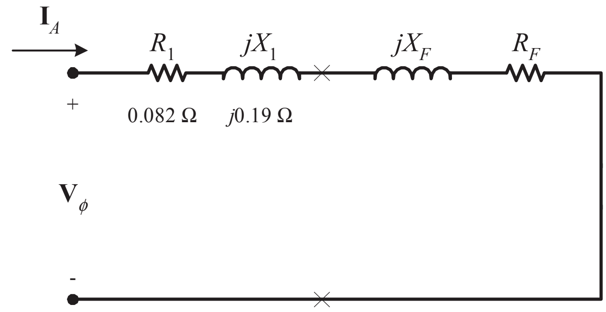
(i) The overall machine efficiency η

(j) The motor speed in revolutions per minute and radians per second

SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\dfrac{1}{jX_M} + \dfrac{1}{Z_2}} = \frac{1}{\dfrac{1}{j7.2\,\Omega} + \dfrac{1}{1.75 + j0.18}} = 1.557 + j0.550 = 1.67\angle 19.2^\circ\ \Omega$$

The phase voltage is 440/$\sqrt{3}$ = 254 V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{254\angle 0^\circ\ \text{V}}{0.082\,\Omega + j0.19\,\Omega + 1.557\,\Omega + j0.550\,\Omega}$$

$$I_L = I_A = 141\angle -24.3^\circ\ \text{A}$$

(b) The stator power factor is

$$\text{PF} = \cos 24.3^\circ = 0.911\ \text{lagging}$$

(c) To find the rotor power factor, we must find the impedance angle of the rotor

$$\theta_R = \tan^{-1}\frac{X_2}{R_2/s} = \tan^{-1}\frac{0.18}{1.75} = 5.87^\circ$$

Therefore the rotor power factor is

$$\text{PF}_R = \cos 5.87^\circ = 0.995\ \text{lagging}$$

(d) The stator copper losses are

$$P_{\text{SCL}} = 3I_A^{\,2}R_1 = 3(141\ \text{A})^2(0.082\ \Omega) = 4890\ \text{W}$$

(e) The air gap power is $P_{\text{AG}} = 3I_2^{\,2}\dfrac{R_2}{s} = 3I_A^{\,2}R_F$

(Note that $3I_A^{\,2}R_F$ is equal to $3I_2^{\,2}\dfrac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2 / s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{\text{AG}} = 3I_2^{\,2}\frac{R_2}{s} = 3I_A^{\,2}R_F = 3(141\ \text{A})^2(1.557\ \Omega) = 92.6\ \text{kW}$$

(f) The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1-s)P_{\text{AG}} = (1-0.04)(92.6\ \text{kW}) = 88.9\ \text{kW}$$

(g) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50\ \text{Hz})}{6} = 1000\ \text{r/min}$$

$$\omega_{\text{sync}} = (1000\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 104.7\ \text{rad/s}$$

Therefore the induced torque in the motor is

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} = \frac{92.6\ \text{kW}}{(1000\ \text{r/min})\left(\dfrac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)} = 884\ \text{N}\cdot\text{m}$$

(h) The output power of this motor is

$$P_{\text{OUT}} = P_{\text{conv}} - P_{\text{mech}} - P_{\text{core}} - P_{\text{misc}} = 88.9\ \text{kW} - 1.3\ \text{kW} - 1.4\ \text{kW} - 300\ \text{W} = 85.9\ \text{kW}$$

The output speed is

$$n_m = (1-s)n_{\text{sync}} = (1-0.04)(1000\ \text{r/min}) = 960\ \text{r/min}$$

Therefore the load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{85.9\ \text{kW}}{(960\ \text{r/min})\left(\dfrac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)} = 854\ \text{N}\cdot\text{m}$$

(i) The overall efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}}\times 100\% = \frac{P_{\text{OUT}}}{3V_\phi I_A\cos\theta}\times 100\%$$

$$\eta = \frac{85.9\ \text{kW}}{3(254\ \text{V})(141\ \text{A})\cos 24.3^\circ}\times 100\% = 87.7\%$$

(j) The motor speed in revolutions per minute is 960 r/min. The motor speed in radians per second is

$$\omega_m = (960\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 100.5\ \text{rad/s}$$